

## Second order difference penalty in action with various tuning

The second order penalty in action for various values of  $\lambda$ . Also shown are the standard deviations of the residuals (*s*) and the roughness measure (*r*). Cubic B-splines, 20 segments. R code in f-d2pen.R

```
# First order difference penalty in action with various tuning
 A graph in the book 'Practical Smoothing. The Joys of P-splines'
#
# Paul Eilers and Brian Marx, 2019
library(ggplot2)
library(gridExtra)
library(JOPS)
# Simulate data
m = 50
set.seed(123)
x = runif(m)
y = sin(2.5 * x) + rnorm(m) * 0.1 + 0.2
# Make basis and penalty
nu = 200
u = seq(0, 1, length = nu)
nseg = 20
Bu = bbase(u, nseg = nseg)
B = bbase(x, nseg = nseg)
nb = ncol(B)
knots = ((1:nb) - 2) / nseg
n = ncol(B)
D = diff(diag(n), diff = 2)
P = t(D) \% D
BtB = t(B) \% \% B
Bty = t(B) \% \% y
# Compute coefficients
A = Mu = NULL
lambdas = c(0.1, 5, 500, 10000)
for (lambda in lambdas) {
  a = solve(BtB + lambda * P, Bty)
  A = cbind(A, a)
```

}

```
Z = Bu %*% A
Mu = cbind(Mu, B \% \% A)
# Generate the plots
plts = list()
for (j in 1:4) {
  # Compute roughness
  aj = c(A[, j])
  da = D %*% aj
  r = sqrt(sum(da ^ 2) / (n - 2))
  r = round(r, 2)
  s = sqrt(mean((y - Mu[, j])^2))
  s = round(s, 2)
  # Scaled basis
  Bsc = B %*% diag(aj)
  # Create data frames for ggplot
  Data = data.frame(x, y)
  Zf = data.frame(x = u, y = Z[, j], id = as.factor(1))
  Bf = data.frame(x = rep(u, n), y = as.vector(Bsc),
                  id = as.factor(rep(1:n, each = nu)))
  Bf$y[Bf$y < 0.0001] = NA
  Bf = na.omit(Bf)
  titl = bquote(lambda==.(lambdas[j]) "|" s==.(s) "|" r==.(r))
  Fa = data.frame(x = knots, y = aj, id = as.factor(1))
  # Build the graphs
  plt1 = ggplot(Data, aes(x = x, y = y), ylim = c(0, 1.5)) +
    geom_point(color = 'grey40') +
    geom_line(data = Zf, aes(x, y), color = "blue", size = 1) +
    geom_point(data = Fa, color = "red", size = 2, shape = 1) +
xlab("") + ylab("") +
    ggtitle(titl) +
    JOPS_theme() +
    theme(plot.title = element_text(size = rel(0.9)))
  # Add to list of plots
  plts[[j]] = plt1
}
# Plot and save
grid.arrange(grobs = plts, nrow = 2, ncol = 2)
```