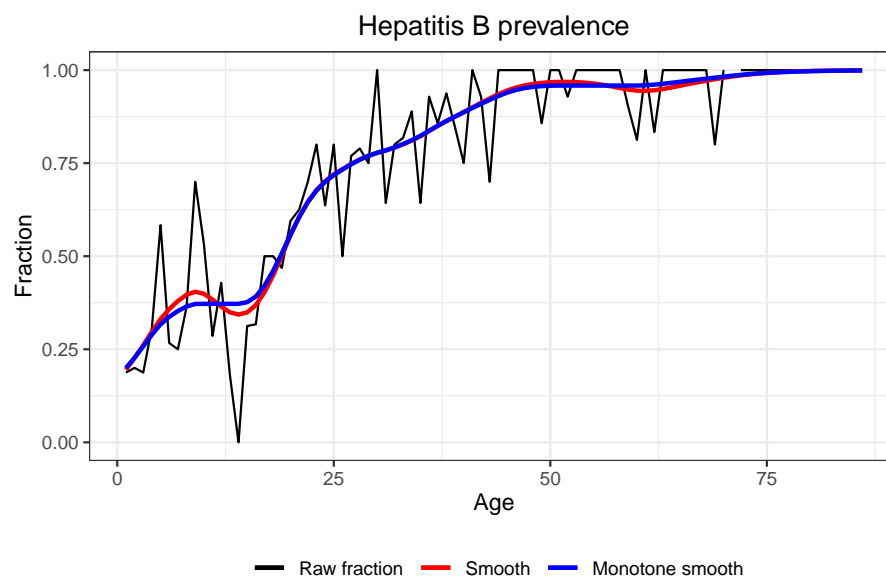


Monotonic smoothing by asymmetric penalty (Hepatitis Bulgaria data)



Generalized linear smoothing of the prevalence of hepatitis B in a sample of Bulgarian males. The raw fractions are shown by the thin black curve. The red curve shows the unconstrained trend. The blue curve is constrained, by an asymmetric penalty, to have a non-negative slope everywhere. R code in `f-hepatitis-mon.R`

```
# Monotonic smoothing by asymmetric penalty (Hepatitis Bulgaria data)
# Data source: Keiding, JRSSA, 1991
# A graph in the book 'Practical Smoothing. The Joys of P-splines'
# Paul Eilers and Brian Marx, 2019

library(ggplot2)
library(JOPS)

# Get the data
data(Hepatitis)
x = Hepatitis$Age
y = Hepatitis$Infected
v = Hepatitis$Sampled
m = length(y)

# Preparations
D = diff(diag(m), diff = 2)
D1 = diff(diag(m))
lambda = 100
P = lambda * t(D) %**% D
kappa = 1e5

# Fit without constraint
p = (y + 1) / (v + 2)
eta = log(p / (1 - p))
u = rep(0, m - 1)
for (it in 1:10) {
  p = 1 / (1 + exp(-eta))
  mu = v * p
  w = mu * (1 - p)
  W = diag(w)
  enew = solve(W + P, y - mu + w * eta)
  dz = max(abs(enew - eta))
}
```

```

    eta = enew
    u = diff(eta) < 0
    if (dz < 1e-4) break
  }
  p1 = p

# Fit with constraint
p = (y + 1) / (v + 2)
eta = log(p / (1 - p))
u = rep(0, m - 1)
for (it in 1:10) {
  p = 1 / (1 + exp(-eta))
  mu = v * p
  w = mu * (1 - p)
  W = diag(w)
  Q = t(D1) %%% diag(u) %%% D1
  enew = solve(W + P + kappa * Q, y - mu + w * eta)
  dz = max(abs(enew - eta))
  eta = enew
  u = diff(eta) < 0
  if (dz < 1e-4) break
}
p2 = p

# Plot results
DF1 = data.frame(x = x, ratio = y / v, p1 = p1, p2 = p2)
plt1 = ggplot(data = DF1) +
  geom_line(aes(x = x, y = ratio, color = 'black')) +
  geom_line(aes(x = x, y = p1, color = 'red'), size = 1) +
  geom_line(aes(x = x, y = p2, color = 'blue'), size = 1) +
  ggtitle('Hepatitis B prevalence') +
  xlab('Age') + ylab('Fraction') +
  scale_color_identity(name = "",
                      breaks = c("black", "red", "blue"),
                      labels = c("Raw fraction", "Smooth", "Monotone smooth"),
                      guide = "legend") +

  theme(
    plot.title = element_text(size=16),
    axis.title.x = element_text(size=16),
    axis.title.y = element_text(size=16)) +
  JOPS_theme() +
  theme(legend.position="bottom")

# Make and save pdf
plot(plt1)

```
